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# CAT Exams

# Must Learn

## By Arun Sharma

LET'S CRACK IT!

# ABOUT THE EDUCATOR

**Arun Sharma**

Arun Sharma is India's most well known author and CAT trainer par excellence.
A graduate from the prestigious Indian Institute of Managament Bangalore, and after a brief stint in the corporate sector, Arun dedicated himself to helping CAT and aptitude test aspirants – from early 1996. Over the past two and a half decades, he has been involved with training students for these exams across North and East India, through his venture Mindworkzz. He is an expert at Quantitative Aptitude, Data Interpretation, Logical Reasoning as well as Verbal Ability, Reading Comprehension and Personality development.

In 2003, after around 8 years of experience in training students, he authored his first book "How to Prepare for Quantitative Aptitude for CAT" which was published by McGraw Hill Education (then TataMcGraw Hill). The book became an overnight best seller and is now in its' eighth edition and has sold over a million copies till date over the past 15+ years. He followed this up with a series of best sellers for CAT (all published by McGraw Hill Education) and his books include – "How to Prepare for Data Interpretation for the CAT", "How to prepare for Logical Reasoning for the CAT", "Teach Yourself Quantitative Aptitude for Competitive exams", "Teach Yourself Data Analysis and Interpretation for Competitive exams". He has also co-authored "How to prepare for Verbal Ability and Reading Comprehension for CAT" with Meenakshi Upadhyay as also "General Studies Paper 2" (McGraw Hill's guide to the Paper 2 of the Civil Services preliminary examination.

An out and out sport enthusiast and a chess and a tennis player, he has the unique record of cracking the CAT 20 times in 20 attempts, starting with his first attempt at CAT in 1992, when he cleared the exam to enter the IIM, Bangalore 1993-95 batch. His normal CAT percentiles have always been above 99.9+, with a score of 99.99 in CAT 2008 and CAT 2018 being his highest scores. He also scored a 100 percentile in DI and a 99.96 percentile in QA in CAT 2018. Ask him why he repeatedly takes the CAT and he has a very simple answer "As an author and a trainer, if I am guiding people to prepare for this exam, I have to be aware of what it takes to be able to solve CAT type questions under pressure. Unless I experience the pressure cooker situation that the actual D-Day brings with it in the CAT, I wouldn't have half insights for training and writing for this exam."

Arun Sharma joined Unacademy as an Educator in 2018 November and has been teaching Plus courses on Data Interpretation and Logical Reasoning for CAT on Unacademy Plus.

# FORMULA SHEET

## Number system

**1. The following operations hold true when you consider even and odd numbers:**

- Even + Even = even
- Odd + Odd = even
- Even + Odd = Odd
- Even any number = even
- Odd x Odd = odd
- Even/Odd = Even (if divisible)
- Odd/Odd = Odd (if divisible)
- Odd/Even = > Never divisible
- Odd + Odd + Odd + Odd+ .... Odd number of times = Odd number

**2. BODMAS Rule :- (Hierarchy of Arithmetic operations)**

**B** - Brackets
**O** - Of
**D** - Division
**M** - Multiplication
**A** - Addition
**S** - Subtraction

**3. Divisibility**

- The following divisibility rules hold true:

**Divisibility by 2 -** If the last Digit of a number is even ie 0, 2, 4, 6 or 8 then the number is divisible by 2.
eg:- 842.

**Divisibility by 3 -** If the sum of all digits is divisible by 3, the number is divisible by 3.

**Divisibility by 4 -** If the last two digits of a number is divisible by 4 or is 00, the number is also divisible by 4.

**Divisibility by 5 -** If the last digit is 0 or 5 the number is divisible by 5
eg:- 125, 525, 220 etc.

**Divisibility by 6 -** A number is divisible by 6 if the number is divisible by both 2 & 3 simultaneously.
Eg:- 144 is divisible by 2 & 3 both hence will also be divisible by 6

**Divisibility by 8 -** If the last three digits of a number is divisible by 8 or have 3 or more zeroes (eg. 000, 0000 etc), the no is also divisible by 8. eg→ 632000.

**Divisibility by 9 -** A number is divisible by 9 if the sum of its digits is a multiple of 9. eg:- 729

**Divisibility by 11 -** If you subtract the sum of digits in the even places from the sum of digits in the odd places (even places and odd places are to be counted from the right), the answer of the subtraction should be either 0 or a multiple of 11. In such a case, the number is said to be divisible by 11. eg:- 1419

- If x is divisible by y, then xz is also divisible by y.
- If x is divisible by y, y is divisible by z then x is divisible by z
- If x is divisible by y, y is divisible by x., then x=y.
- If x is divisible by a, y is divisible by a then x+ y and x- y are both divisible by a.
- If a is divisible by c and b is divisible by d, then ab is divisible by cd.
- The highest power of a prime number 'p', which divide x! exactly is given by:

$$\left[\frac{x}{p}\right] + \left[\frac{x}{p^2}\right] + \left[\frac{x}{p^3}\right] + \dots$$

## 4. Units Digit

- Cyclicity

| When The Last Digit Is | Cyclicity Is | Unit Digit Cycle | | | |
|---|---|---|---|---|---|
| 0 | 1 | Always 0 at any power | | | |
| 1 | 1 | Always 1 at any power | | | |
| 5 | 1 | Always 5 at any power | | | |
| 6 | 1 | Always 6 at any power | | | |
| 4 | 2 | 4 at $4^{2n+1}$ | 6 at $4^{2n}$ | | |
| 9 | 2 | 9 at $9^{2n+1}$ | 1 at $9^{2n}$ | | |
| 2 | 4 | Same as $2^1$ at $2^{4n+1}$ | Same as $2^2$ at $2^{4n+2}$ | Same as $2^3$ at $2^{4n+3}$ | Same as $2^4$ at $2^{4n}$ |
| 3 | 4 | Same as $3^1$ at $3^{4n+1}$ | Same as $3^2$ at $3^{4n+2}$ | Same as $3^3$ at $3^{4n+3}$ | Same as $3^4$ at $3^{4n}$ |
| 7 | 4 | Same as $7^1$ at $7^{4n+1}$ | Same as $7^2$ at $7^{4n+2}$ | Same as $7^3$ at $7^{4n+3}$ | Same as $7^4$ at $7^{4n}$ |
| 8 | 4 | Same as $8^1$ at $8^{4n+1}$ | Same as $8^2$ at $8^{4n+2}$ | Same as $8^3$ at $8^{4n+3}$ | Same as $8^4$ at $8^{4n}$ |

Numbers and sum of factors:

$X = a^m \times b^n \times c^o$

Where a, b and c are prime factors of X.
Number of factors = (m+ 1) (n+ 1) (o+ 1).
Sum of all the factors = .

$(a^0 + a^1 + a^2 + \ldots + a^m)(b^0 + b^1 + b^2 + \ldots + b^n)(c^0 + c^1 + c^2 + c^3 + \ldots + c^0)$

**5. Some important algebraic formulae used in Number system**

$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
$(a - b)^3 = a^3 - b^3 - 3ab(a + b)$
$a^2 - b^2 = (a + b)(a - b)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
If, $a + b + c = 0$ then, $a^3 + b^3 + c^3 - 3abc = 0$

## Progressions

### a. Arithmetic Progression

1. $n^{th}$ TERM OF A.P.

$n^{th}$ term = $a_n$ = any term value + (n – term number) × d

Corresponding

e.g. If first term value is know, then

$a_n = a_1 + (n - 1)d$ since term number = 1 (refer, assumption)

COMMON DIFFERENCE (d)

$$\frac{\text{Difference of two term values}}{\text{Difference of two corresponding term numbers}}$$

$$d = \frac{a_2 - a_1}{2-1} = \frac{a_x - a_3}{x-3} = \ldots = \frac{a_q - a_p}{q-p} = \ldots = \frac{a_n - a_{n-1}}{n-(n-1)} = \ldots = \frac{a_n - a_1}{n-1}$$

SUM UP TO $n^{th}$ TERM ($S_n$)

Sum up to $n^{th}$ term = $S_n$ = 1 $^{st}$term + 2 $^{nd}$term + ..... + n $^{th}$term

$$S_n = \frac{n}{2}[\,2.\text{any term value} + (n+1-2\,.\,\underbrace{\text{term number}}_{\text{Corresponding}})\,.\,d\,]$$

e.g. If only the value of the first term is known, then term number = 1, and term value = $a_1$

[Refer, assumption]

$S_n = n/2\,[\,2a_1 + (n-1)d]$

**b. Geometric Progression:**

Assumption : The given geometric sequence is

$G_1$ $G_2$ $G_3$ ... $G_{N-1}$ $G_N$

$$\text{Common Ratio} = r = \frac{G_2}{G_1} = \frac{G_3}{G_2} = \ldots = \frac{G_n}{G_{n-1}}$$

nth term of a G.P. = First term × (common ratio)$^{n-1}$
$G_n = G_1 \times (r)^{n-1}$

Sum of n terms of the geometric series= $G_1(r^n-1)/r-1$ (If r> 1) or $G_1(1-r^n)/1-r$ (If r< 1)

Sum of the infinite series of the geometric series= $G_1/1-r$

**c. Harmonic Progression:**

a, b, c, ..... are said to be in harmonic progression when 1/a, 1/b, 1/c, ... are in arithmetic progression.

Harmonic mean: If A and B are two quantities and H is their harmonic mean then:

$$H = \frac{2AB}{A+B}$$

**Some useful results:**

1. Sum of 1st 'n' natural numbers= $\frac{n(n+1)}{2}$

2. Sum of the squares of the first 'n' natural numbers= $\frac{n(n+1)(2n+1)}{6}$

3. Sum of cubes of first 'n' natural numbers= $\left(\frac{n(n+1)}{2}\right)^2$

4. Sum of first 'n' odd natural number= $1+3+5+....+(2n-1)=n^2$

5. Sum of first 'n' even natural number= $2+4+6+8+....+2n=n^2+n$

## Average:

1. The basic formula for average of n numbers $a_1, a_2, a_3, .....,an$ is $= (a_1+a_2+a_3+.....+a_n)/n$

2. If the average of n1 numbers is A1, n2 numbers is A2 and so on then the Weighted average= $(n_1A_1+n_2A_2+n_3A_3+.....+n_kA_k)/(n_1+n_2+n_3+.....+n_k)$

## Alligations:

1. In the specific case when two groups are being mixed, there is an alternate process to think of the weighted average. This process is called allegation and can be explained as follows:

If we start from the weighted average equation for the case of two groups being mixed:
$Aw = (n_1A_1 + n_2A_2) / (n_1 + n_2)$
Rewriting this equation we get : $(n_1 + n_2)\,Aw = n_1A_1 + n_2A_2$
$n_1(Aw - A_1) = n_2(A_2 - Aw)$
or, $n_1/n_2 = (A_2 - Aw)/(Aw - A_1)$ →**The alligation equation.**

2. Amount of original liquid left after *R* operations when the vessel initially contains *N* litres of liquid and from which *M* units are taken out each time = $N(1-M/N)^R$

## Percentages:

1. Percentage change = $\frac{\text{Absolute value changes}}{\text{Original Quantity}} \times 100$

2. If we change the quantity first by a% then by b% then net percentage change = $a + b + \frac{ab}{100}$ (we use '+' sign for the increase and '-' sign for the decrease).

## Profit & Loss:

1. Selling price = $\left(1 + \frac{\%\ Gain}{100}\right) \times C.P$ or $\left(1 - \frac{\%\ loss}{100}\right) \times C.P$

2. Loss = cost price- selling price

3. Profit= Selling price- Cost price

4. %Profit or % Loss= $\frac{\text{Net profit or net loss}}{\text{Cost price}} \times 100$

5. Break-even sales is defined as the volume of the sale at which there is no profit no loss.

6. % Profit= (Goods left after achieving break even sales X 100)/Goods sold

7. CP+ markup = Marked Price

8. CP+ %Mark-up on CP = Marked price

9. Selling Price = Marked Price – Discount

10. Selling Price = Marked Price - (%discount on Marked Price)

11. When two articles are sold at the same price but one of them at a profit and another at a loss and the percentage profit is the same as the percentage loss .in this case there is always a loss. Loss %= $(\text{common gain or loss}/10)^2$

## Interest:

**P: Principal, t= time, r= rate**

1. Simple Interest= (P x t x r)/100

2. Compound Interest= $P\left(1 + \frac{r}{100}\right) - P$ Total amount after t years = $P\left(1 + \frac{r}{100}\right)^t$

If the interest is compounded half yearly, then the compound interest= $P\left(1 + \frac{r/2}{100}\right)^{2t} - P$

If the interest is compounded quarterly, then the compound interest= $P\left(1 + \frac{r/4}{100}\right)^{4t} - P$

3. The difference between the compound interest and simple interest over two years=

$$P\left(\frac{r}{100}\right)^2$$

## Ratio & Proportion:

1. $\frac{a}{b} = \frac{ma}{mb}$

2. $\frac{a}{b} = (\frac{a}{d}) / (\frac{b}{d})$

3. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k = \frac{a+c+e}{b+d+f}$

4. If a/b, c/d, e/f, ... are unequal fraction then $\frac{a+c+e+....}{b+d+f+....}$ lies between the highest and the lowest of these fractions

5. If $\frac{a}{b} > 1$ , then $\frac{a+k}{b+k} < \frac{a}{b}$ and $\frac{a-k}{b-k} > \frac{a}{b}$

6. If $\frac{a}{b} < 1$ , then $\frac{a+k}{b+k} > \frac{a}{b}$ and $\frac{a-k}{b-k} < \frac{a}{b}$

7. (a+c)/(b+d) > (a/b) if c/d>a/b

   (a+c)/(b+d) < (a/b) if c/d<a/b

**8. Some facts about proportion**

**a. Inverted**

If a/b=c/d  b/a=d/c

**b. Alternant**

If a/b=c/d a/c=b/d

**c. Componendo**

If a/b=c/d=a+b/b=c+d/d

**d. Dividendo**

If a/b=c/d=a-b/b=c-d/d

**e. Componendo and dividendo**

If a/b=c/d (a+b/a-b)=(c+d/c-d)

## Time and Work:

1.

$(M_1 \times D_1 \times H_1) / (M_2 \times D_2 \times H_2) = W_1 / W_2$

Where

$M_1$ = No. of persons required to do $W_1$ work

$D_1$ = No. of days required to do $W_1$ work

$H_1$ = No of hours per day being worked by $M_1$ persons

$W_1$ = Work done by $M_1$ persons and

$M_2$ = No. of persons required to do $W_2$ work

$D_2$ = No. of days required to do $W_2$ work

$H_2$ = No of hours per day being worked by $M_2$ persons

$W_2$ = Work done by $M_2$ persons

2. If A can do a piece of work in n days, then A's 1 day's work will be = 1/n

3. If A is thrice as good a workman as B then, the ratio of work done by A to that of B will be = 3:1 and the ratio of the times taken by A and B to finish a work will be 1:3.

## Time, speed and distance:

1. Distance (S) = Speed (V) × Time (T)

2. If two vehicles A and B start at the same time from points P and Q towards each other and after meeting each other, they take m and n hours to reach their respective ends, then

A's speed: B's speed = (n/m) ½

**3. TRAINS and BOATS:**

Concept of Relative speed: When two bodies are moving towards each other or moving away from each other, the absolute speed with which they are coming towards each other or going away from each other is known as their Relative speed.

**When they are moving in the same direction :**

a. In case of trains: If the speed of the first train is a km/hr and that of the other train is b km/hr, then the relative speed of trains = (a – b) km/hr; a>b

b. In case of boats: If the speed of the boat is a km/hr and that of the stream is b km/hr, then the relative speed = (a + b) km/hr

By definition, this kind of movement is known as Downstream movement and the relative speed in this case is called Downstream Speed.

**When they are moving in the opposite direction :**

a. In case of trains: If the speed of first the train is a km/hr and that of the other train is b km/hr, then the relative speed of trains = (a + b) km/hr

b. In case of boats: If the speed of the boat is a km/hr and that of the stream is b km/hr, then the relative speed = (a - b) km/hr; a > b

By definition, this kind of movement is known as Upstream movement and the relative speed in this case is called Upstream Speed.

**4. Special Cases for Trains:**

a. When a train passes a pole (or, any stationary object of negligible length), it covers a distance which is equal to its own length.

b. When a train passes a platform, it covers a distance which is equal to the sum of the length of the platform and its own length.

c. When a train A passes a moving train B, it covers a distance which is equal to the sum of the length of both the trains A and B with the relative speed as given earlier.

d. When a train A crosses a stationary train B, it covers a distance which is equal to the sum of the length of both the trains.

e. When a train passes a person sitting on the window seat in another moving train, the train covers a distance equivalent to its own length with the relative speed as given earlier..

**5. Special Cases for Boats:**

Speed of boat is a km/hr and the speed of stream is b km/hr, (where a>b).
Speed of boat = ½(Downstream speed + Upstream speed)
Speed of stream = ½(Downstream speed - Upstream speed)

In the case of boats and streams, as the distance is constant in upstream and downstream movements, time taken is inversely proportional to the upstream and downstream speeds.

**6. Races**

**a.** If A gives a start of 10 meters to B – It means that A starts running after B has covered a distance of 10m.

**b.** If A gives a start of 10 seconds to B - It means that A starts 10 seconds after B has started running.

**c.** Race ends in a dead-heat – It means that race ends in a tie or both A and B end the race at the same time.

**7.** To convert speed in km/ph to m/sec, multiply it with 5/18. To convert speed in m/sec to km ph, multiply it with 18/5.

**8.** When the same distance is traveled at two different speeds say $S_1$ and $S_2$, the average speed would be equal to: $2S_1S_2/(S_1+S_2)$

Let the two people A and B with respective speeds of SA and SB (SA>SB) be running around a circular track (of length C) starting at the same point at the same time. Then

a.

| Time | When the two persons are running in the SAME direction | When the two persons are running in OPPOSITE directions |
|---|---|---|
| Time t Time taken to meet for the FIRST TIME EVER | $C/(S_A - S_B)$ | $C/(S_A + S_B)$ |
| Time t Time taken to meet for the first time at the STARTING POINT | LCM of times taken by each to complete 1 round<br>viz: LC<br>LCM of (C/SA, C/SB) | LCM of time taken by each to complete 1 round |

**b. When three people are running around a circular track in the same direction**

Let the three people A, B, C be running around a circular track of circumference C in the same direction with respective speeds of $S_1$, $S_2$ ,$S_3$ where $S_1>S_2>S_3$. They start from the same point on the circle at the same time

| | |
|---|---|
| Time taken to meet for the first time ever | LCM of times taken for the fastest runner to completely overtake each of the slower runners. In other words, if we define the times $S_{AB}$ and $S_{AC}$ as the respective time taken by the fastest runner A to completely overlap B and C respectively. Then the time for the first meeting at any point of the circle would be the LCM $S_{AB}$. and $S_{AC}$. |
| Time taken to meet for the first time at the starting point | LCM of times taken by each to completeone full round of the circle. |

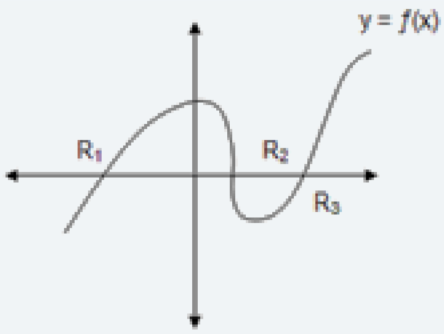
## Functions, equations, inequalities and logarithm:

1. A function is of the form $y = f(x) < 0$ or $f(x) \leq 0$, While an equation is ofthe form $f(x) = 0$.

2. A function yields a locus of points (x, y) which can be plotted on the x-yaxis, An inequality yields a range of values for x at which the inequality issatisfied while an equation yields a solution or multiple solutions dependingon the nature of $f(x)$ [Note: solutions of equations are also called as roots].

3. Typically $f(x)$ can represent line or, quadratic, cubic or larger expressions – thus giving rise to linear function, linear equations & linear inequalities. Likewise, we can have quadratic or cubic function, quadratic or cubic equations (having 2 or 3 solutions/roots).

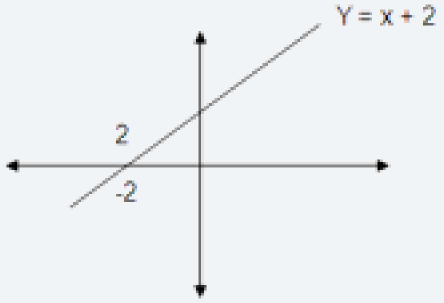
4. The relationship between a function, equation & inequality can be visualized below.



In the above function, $R_1$, $R_2$, & $R_3$ are the roots of the equation $f(x) = 0$, while the inequality $f(x) \geq 0$ is satisfied at $R_1 \leq x \leq R_2$ & $R_3 \leq x$

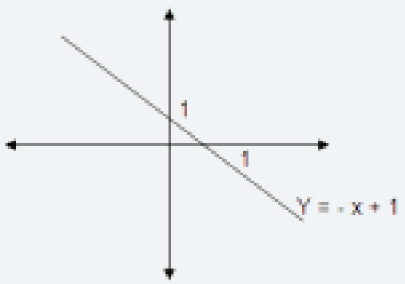
In general, the solutions/roots of the equation $f(x) = 0$ are visible at the points where the graph of $y = f(x)$ cuts the x – axis.

5. Thus, linear functions are straight lines because they cut the x-axis only once.




Linear function with positive coefficient of x.

b.




Linear function with negative coefficient of x.

6. Quadratic functions cut the x – axis twice (Because quadratic yields two roots).

7. The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

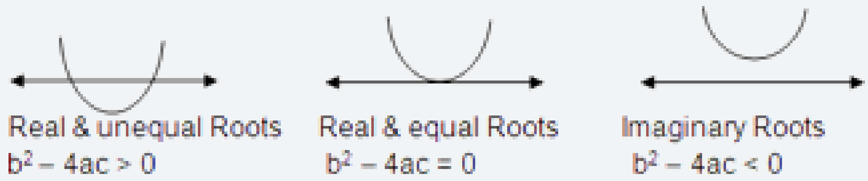
$$R_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$R_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

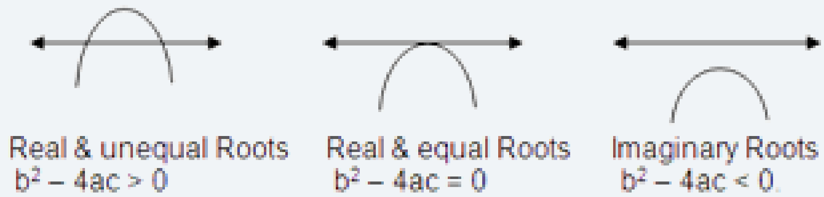
Based on whether $b^2 - 4ac$ is positive (Real Roots which are unequal), 0(Real & equal roots) or negative (Imaginary roots) the nature of the roots of the quadratic equation varies. Hence, $b^2 - 4ac$ is called the Discriminant of quadratic equation.

9. The following are the possible graphs for quadratic function.

a. If the coefficient of $x^2$ is positive




b. If the coefficient of $x^2$ is negative. (i.e. a is negative)



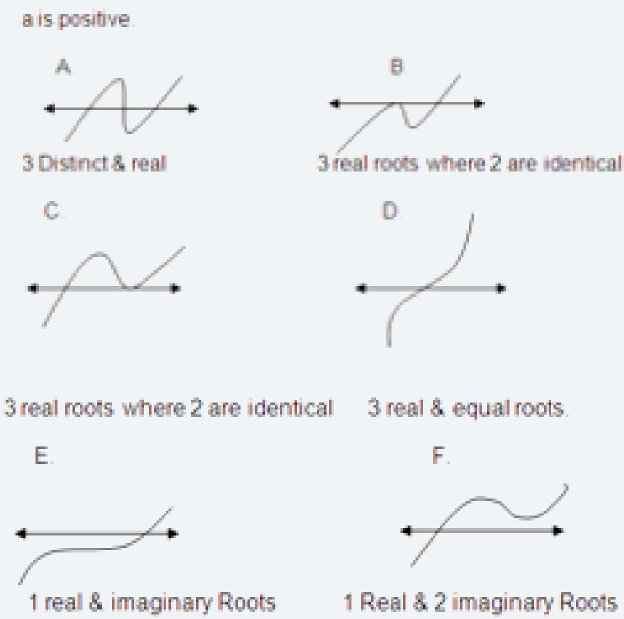

10. The sum of roots of a quadratic equation R1 + R2 = -b/a Product of Roots = c/a.

11. Roots of quadratic equation are of opposite sign means that their product is negative.

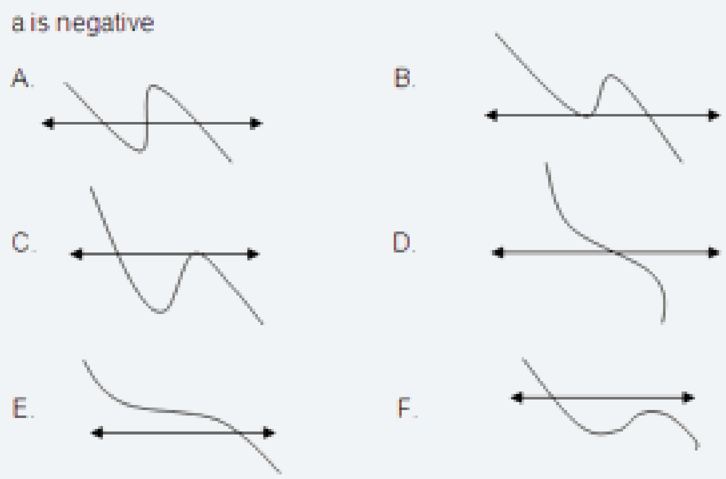
12. Cubic function cut the x – axis thrice (As cubic equations should yield three roots).

13. Standard cubic function $ax^3 + bx^2 + cx + d$

a.



b.




Note: cubic function can have 3 real & equal, 3 real & distinct 3 real where 2 equal or 1 real & 2 imaginary roots.

14. For the cubic equation $ax^3 + bx^2 + cx + d = 0$

    Product of roots = -d/a = R1 R2 R3

    Sum of roots = -b/a = R1+ R2+R3

    Product of roots taken two at a time= R1R2+ R2R3+R3R1= c/a

15. Similarly we can visualize functions with $x^4$, $x^5$ so on.

16. **Rules of logarithms:**

a) Logarithm to a negative base is not defined.

b) Logarithm of a negative number is not defined. Hence, in any logarithm equation, $\log_a m = x$ , and we can say that m>0 and a>0.

c) ax =m

→ x= $\log_a m$

And $\log_a m = x$ → $a^x$=m

In short, $a^x$ =m → x=$\log_a m$.

x=logam is called the logarithmic form and ax=m is called the exponential form of the equation connecting a, x and m.

d) Since logarithm of a number is a value, it will have an integral part and a decimal part. The integral part of the logarithm of a number is called the CHARACTERISTIC and the decimal part of the logarithm is called the MANTISSA.

e) Logarithm can be expressed to any base.

f) Logarithm from one base can be converted to logarithm to any base

g) Natural logarithm or Napierian logarithm: These are logarithm expressed to the base of a number called "e".

h) Common logarithm: These are logarithm expressed to the base 10. For most of the problems under logarithm. It is common logarithm that we deal with. In examinations also, if logarithms are given without mentioning any base, it can normally be taken to be logarithms to the base 10

## POINTS REGARDING COMMON LOGARITHM:

a) The characteristic of the logarithm of a number greater than unity is positive and is less by one than the number of digits in its integral part.

b) The characteristic of the logarithm of a number less than one, is negative and its magnitude is one more than the number of zeroes immediately after the decimal point.

c) The mantissas are the same for the logarithms of all numbers which have the same significant digits.

**Given below are some important rules**

a) $\log_a 1=0$ (log of 1 to any base is 0).

b) $\log_a a=1$ (logarithm of any number to the same base is 1)

c) $\log_a(mn)= \log_a m+ \log_a n$

d) $\log_a(m/n)= \log_a m – \log_a n$

e) $\log_a mp=px \log m$

f) $\log_a b=1/\log_b a$

g) $\log_a m= \log_b m/\log_b a$

h) $\log_{a^q} (m^p) = p/q \log_a m$

**Inequalities:**

a) a> b, b> c then a> b> c.

b) a> b, then for any c, a+ c> b+ c. in other words an equality remains true if the same number added on both sides of the equality.

c) If $a > b$ and $c > 0$ then, $ac > bc$. If $c < 0$, then $ac < bc$.

d) If $a > b$, $c > d$, then $a + c > b + d$

e) If $a > b$, $c < d$, then $a - c > b - d$.

f) If a and b are natural numbers and $a > b$, then, $a^n > b^n$

g) $|a+b| \le |a| + |b|$ & $|a-b| \le |a| - |b|$

h) $a^3+b^3 \ge ab(a+b)$ if a and b both are greater than 0 and equality hold when $a = b$.

i) $a^2+b^2+c^2 \ge ab+bc+ac$

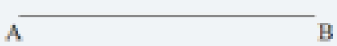
j) $\frac{a+b}{2} \ge (ab)^{1/2}$

## Geometry and Mensuration:

**1. Lines and their properties**

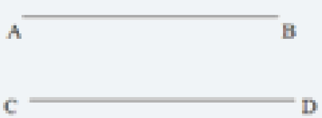
A line is a set of points placed together that extends into infinity in both directions

Straight lines




**(a)Parallel lines:**

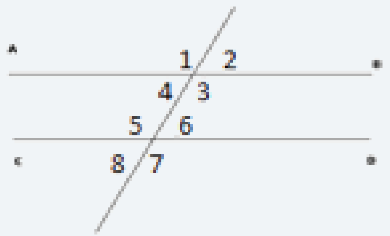
Two straight lines are parallel if they lie on the same plane and do not intersect however far produced.




**(b)Transversal:**

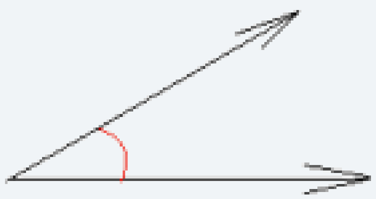
it is a straight line that intersects two parallel lines then:

a. Corresponding angles are equal. Ex: $\angle 1 = \angle 5, \angle 2 = \angle 6$.
b. Alternate interior angles are equal. Ex: $\angle 4 = \angle 6, \angle 3 = \angle 5$.
c. Alternate exterior angles are equal. Ex: $\angle 1 = \angle 7, \angle 2 = \angle 8$.
d. Interior angles on the same side of transversal add up to 1800. $\angle 4 + \angle 5 = 180^0$.



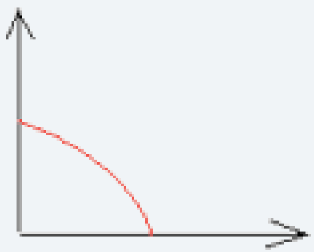

**Acute angle:**

An angle whose measure is less than 90 degrees. The following is an acute angle.



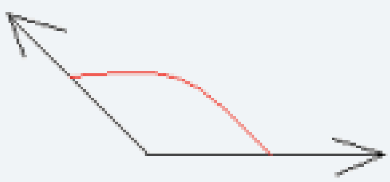
**Right angle:**

An angle whose measure is 90 degrees. The following is a right angle.



**Obtuse angle:**

An angle whose measure is bigger than 90 degrees but less than 180 degrees. Thus, it is between 90 degrees and 180 degrees. The following is an obtuse angle.



**Straight angle:**

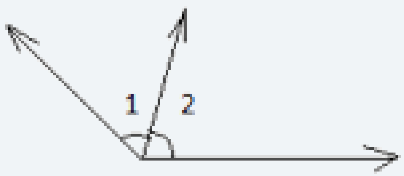
An angle whose measure is 180 degrees.Thus, a straight angle look like a straight line. The following is a straight angle.

**Reflex angle:**

An angle whose measure is bigger than 180 degrees but less than 360 degrees. The following is a reflex angle.

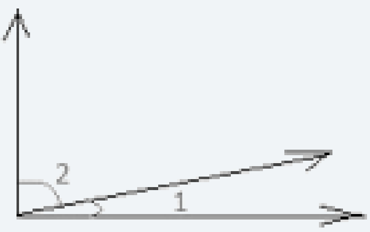
**Adjacent angles:**

Angle with a common vertex and one common side. <1 and <2, are adjacent angles.



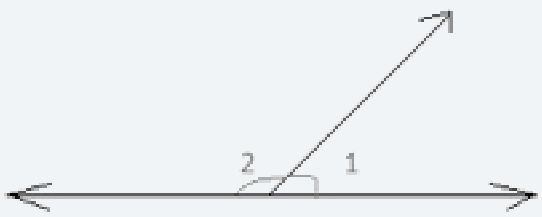

**Complementary angles:**

Two angles whose measures add to 90 degrees. Angle 1 and angle 2 are complementary angles because together they form a right angle.
Note that angle 1 and angle 2 do not have to be adjacent to be complementary as long as they add up to 90 degrees



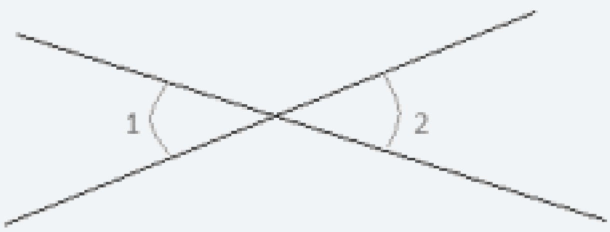
**Supplementary angles:**

Two angles whose measures add to 180 degrees. The following are supplementary angles.

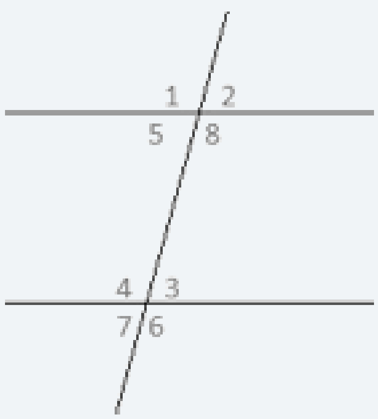



**Vertical angles:**

Angles that have a common vertex and whose sides are formed by the same lines. The following(angle 1 and angle 2) are vertical angles.




When two parallel lines are crossed by a third line(Transversal), 8 angles are formed. Take a look at the following figure.




Angles 3,4,5,8 are interior angles

Angles 1,2,6,7 are exterior angles

**Alternate interior angles:**

Pairs of interior angles on opposite sides of the transversal.

For instance, angle 3 and angle 5 are alternate interior angles. Angle 4 and angle 8 are also alternate interior angles

**Alternate exterior angles:**

Pairs of exterior angles on opposite sides of the transversal.

Angle 2 and angle 7 are alternate exterior angles.

**Corresponding angles:**

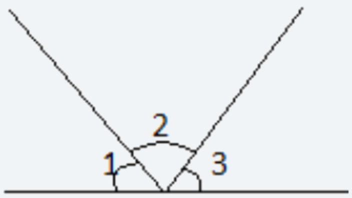
Pairs of angles that are in similar positions.

Angle 3 and angle 2 are corresponding angles.

Angle 5 and angle 7 are corresponding angles

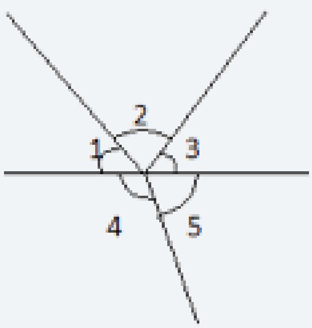
**Angles on the side of a line**

$$\angle 1 + \angle 2 + \angle 3 = 180^o$$




**Angles of round the point**

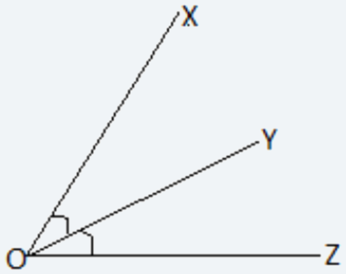
$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360^o$$



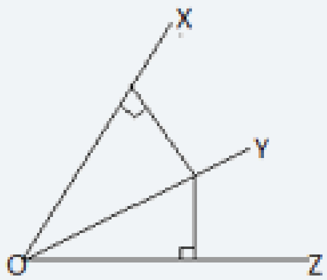
**Angle Bisector:**

OC is the angle bisector ∠ XOZ

i.e.,$\angle XOY = \angle ZOY = \dfrac{1}{2}\angle XOZ$

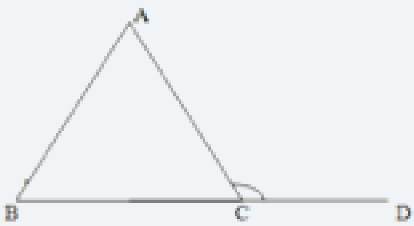



When a line segment divides an angle equally into two parts, then it is said to be the angle bisector (OZ) (Angle bisector is equidistant from the two sides of the angle)

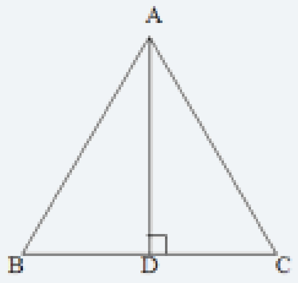



## Triangles:

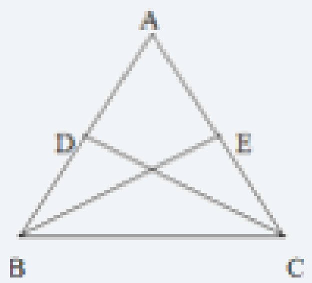
- Sum of the three angles of a triangle is $180^0$
- The exterior angle of triangle (at each vertex) is equal to the sum of the two opposite interior angles.




- The perpendicular drawn to a side from the opposite vertex is called the altitude to that side

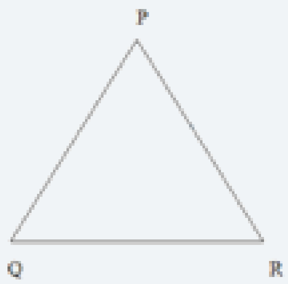



- The line joining the midpoint of aside with the opposite vertex is called the median drawn to that side. A median divides the triangle into two equal halves as far as the area is concerned.



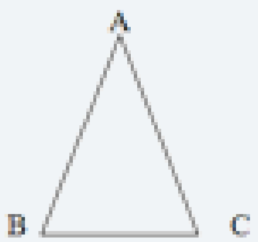

**(a) Equilateral triangle**

An equilateral triangle is one in which all the sides are equal (and hence, all angles are equal, i.e. each of the angles is equal to $60^0$).




**(b) Isosceles triangle**

An isosceles triangle is one in which two sides are equal (and hence, the angles opposite to them are equal).




Here, AB = AC

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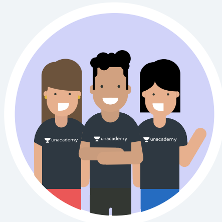

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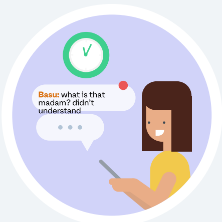


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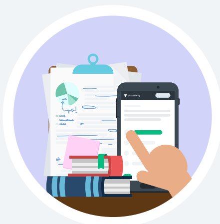

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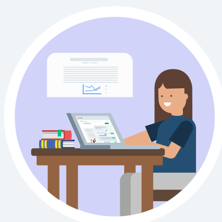

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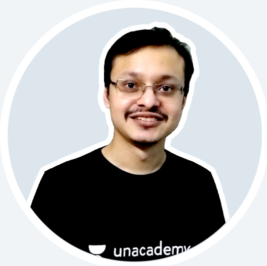

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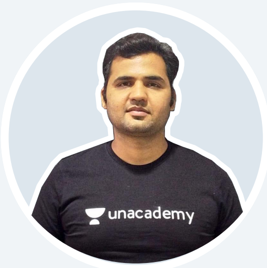

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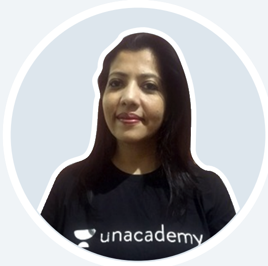

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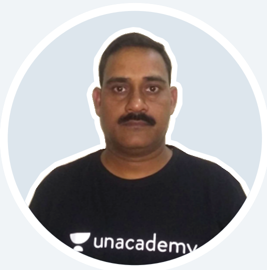

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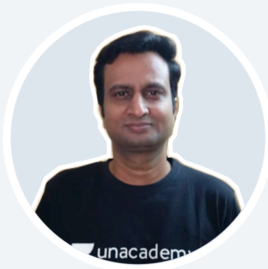

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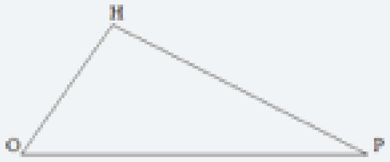
**(b) Isosceles triangle**

A scalene triangle is one in which no two sides are equal.




**(d) Pythagorean Triplet**

A Pythagorean triplet is a set of three positive whole numbers a, b and c that are the lengths of the sides of a right triangle.

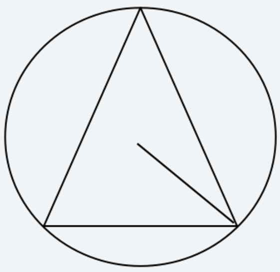
$a^2+b^2=c^2$.

**(e) Common Pythagorean Triplets:**

3,4,5; 5,12,13; 7,24,25; 8,15,17; 9,40,41 ; 11,60,61; 12,35,37 etc

**(f) Circumcentre**

Circumcentre is the point of intersection of the three perpendicular bisectors of the a triangle.



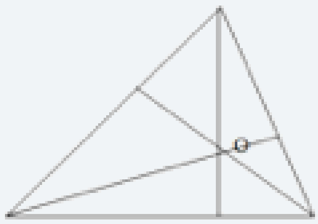
The circumcentre of a right angled triangle is the midpoint of the hypotenuse of a right –angled triangle.

**(g) Incentre**

The internal bisectors of the three angles of a triangle meet at a point called incentre of the

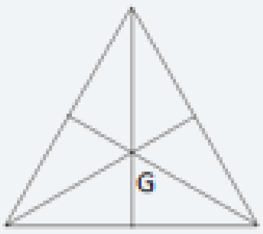
**(h) Orthocenter**

The three altitudes meet at a point called orthocenter.




**(i) Centroid**

The three medians of a triangle meet at a point called the centroid and it is represented by G.




**(j) Similarity of triangles**

Two triangles are said to be similar if the here angles of one triangle are equal to the three angles of the second triangles. Similar triangles are alike in shape only the corresponding angles of two similar triangles are equal but the corresponding sides are only proportional

Two triangles are similar if,

- The three angles of one are respectively equal to the three angles of the second triangle.
- Two sides of one triangle are proportional to two sides of the other and the included angles are equal.

In two similar triangles

Ratio of sides = ratio of heights = Ratio of the length of the medians = Ratio of the length of the angular Bisectors = Ratio of inradii = Ratio of circumradii

Ratio of areas =Ratio of squares of corresponding sides.

**(k) Congruency of triangles**

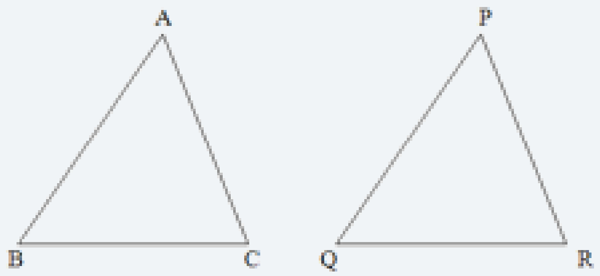
Two triangles are identical in all respects are said to be congruent.

In two congruent triangles

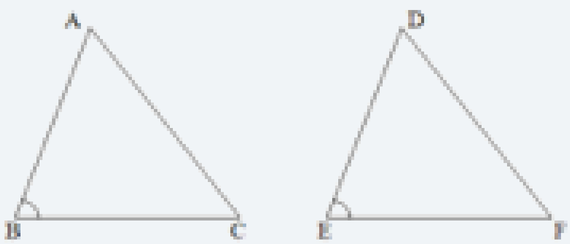
- The corresponding sides (i.e., sides opposite to equal angles) are equal. The corresponding angles (angle opposite to equal sides) are equal

Two triangles will be congruent if at least one of the following conditions is satisfied:

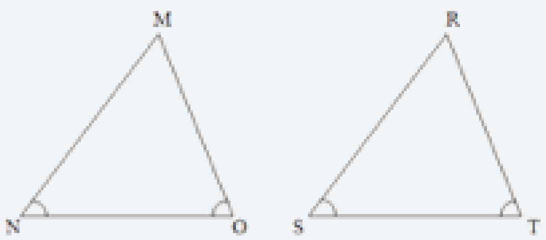
- Three sides of one triangle are respectively equal to the three sides of the second triangle (side-side-side congruency).




- Two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the second triangle (normally referred to as side-angle-side congruency).




- Two angles and one side of a triangle are respectively equal to two angles and the corresponding side of the second triangle (normally referred to as the a-s-a rule, i.e. angle-side-angle congruency).




- For any triangle three sides of the measurements a, b, c are given.

  Area=$[s(s-a)\ (s-b)\ (s-c)]^{1/2}$

  Where s=a+b+c/2

- When base and altitude (height) to that base are given
  Area=1/2 × base × altitude= 1/2.b.h

- Area=1/2 ab  sinC =1/2 bc.sinA =1/2 ca.sin B

- Area=ABC/4R  where R is the circumradius of the triangle

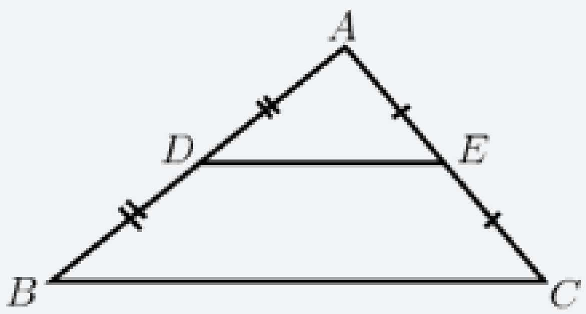
- Area = r.s
  Where r is the inradius of the triangle and s, the semi perimeter

- For an isosceles triangle

  Area = (b/4) $\sqrt{4a^2 - b^2}$ where a is length of each of the two equal sides and b is the third side.

**Mid-Point Theorem**

The line segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half the third side.

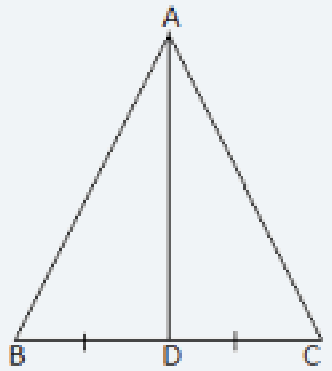



AD = BD and AE = CE
DE ||BC

**Apollonius' theorem**

"The sum of the squares of any two sides of any triangle equals twice the square on half the third side, together with twice the square on the median bisecting the third side"

Specifically, in any triangle ABC, if AD is a median, then



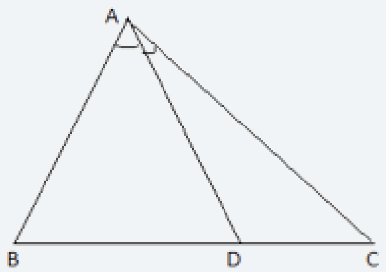

BD = CD
AD is the median

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

**Angle bisector theorem**

In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. i.e.,

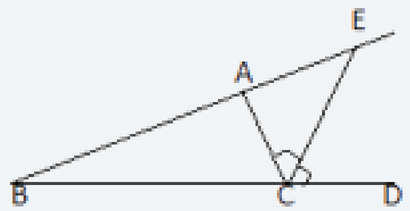
$$\frac{BD}{CD} = \frac{AB}{AC} \text{ and } BD \times AC - CD \times AB = AD^2$$




**Exterior angle bisector theorem**

In a triangle the angle bisector of any exterior angle of a triangle divided the side opposite to the external angle in the ratio of the remaining two sides i.e,

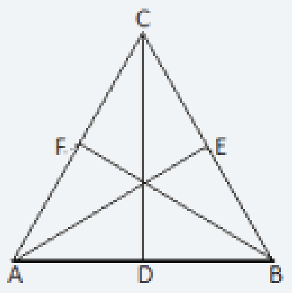
$$\frac{BE}{AE} = \frac{BC}{AC}$$



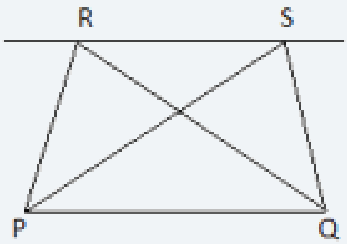
## Some Useful Results

1. In a triangle AE, CD and BF are the medians then

$3(AB^2 + BC^2 + AC^2) = 4(CD^2 + BF^2 + AE^2)$

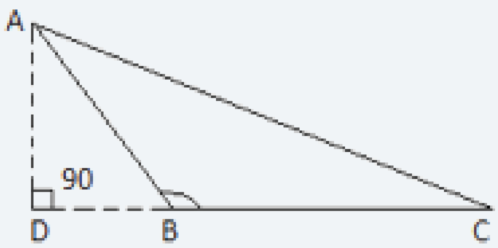



2. If the two triangles have the same base and lie between the same parallel lines (as shown in figure), then the area of two triangles will be equal.



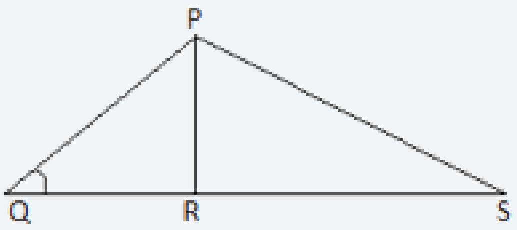

i.e., $A(\Delta ABC) = A(\Delta ADB)$

3. In an obtuse angle $\Delta$ABC, AD is perpendicular dropped on BC. BC is produced to D to meet AD, then



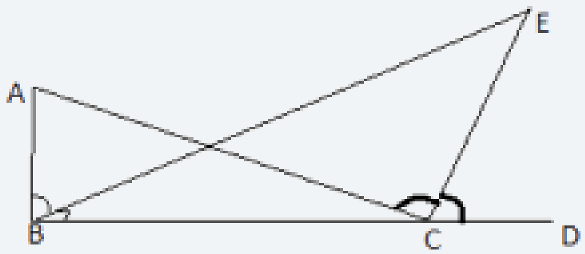

$AC^2 = AB^2 + BC^2 + 2BD \cdot BC \qquad (\angle B > 90^0)$

4. In an acute angle $\Delta$ PQS , PR is perpendicular dropped on the opposite side of $\angle$ P, then



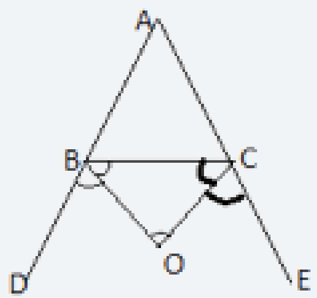

$PS^2 = PQ^2 + QS^2 - 2QR \cdot RS \quad (\angle Q < 90^0)$

5. In a $\triangle$ ABC , if side BC is produced to D and bisectors of $\angle$ ABC and $\angle$ ACD meet at E, then



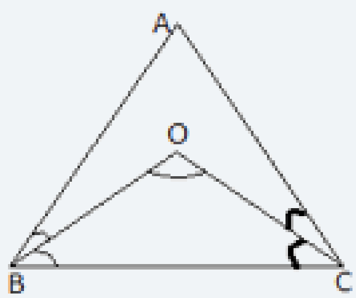

$\angle BEC = 1/2 \angle BAC$

6. In a $\triangle$ ABC , if side AB and AC are produced to D and E respectively and the bisectors of $\angle$DBC and $\angle$ECB intersect at O, then



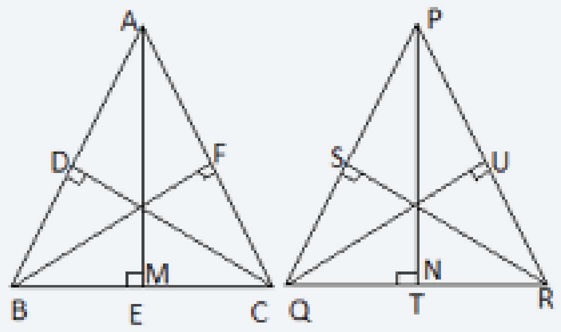

$\angle BOC = 90^0 - 1/2 \angle A$

7. In a $\triangle$ ABC , if the bisectors of $\angle$B and $\angle$C meet at O then $\angle BOC = 90^0 + 1/2 \angle A$



### Properties of similar triangles

If the two triangles are similar, then for the proportional/corresponding sides we have the following results




1. Ratio of sides = Ratio of heights (altitudes)
   = Ratio of medians
   = Ratio of angle bisectors
   = Ratio of inradii
   = Ratio of circumradii

2. Ratio of areas = Ratio of square of corresponding sides.

   ie., if $\triangle$ ABC ~ $\triangle$ PQR, then

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

## QUADRILATERALS

### a) Parallelograms

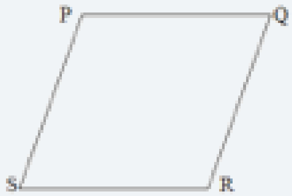
A quadrilateral in which opposite sides are parallel is called a parallelogram.

In a parallelogram

Opposite sides are equal
Opposite angles are equal
Each diagonal divides the parallelogram into two congruent triangles.
Sum of any two adjacent angles is $180^0$.
The diagonals bisect each other.

**(b) Rhombus**

A parallelogram having all the sides equal is a rhombus.




Area = 1/2×product of diagonals × sine of the angle between them
$= 1/2 \times d_1 \times d_2 \sin 90^0$
$= 1/2 \times d_1 \times d_2$

Area = product of adjacent sides × sine of the angle between them

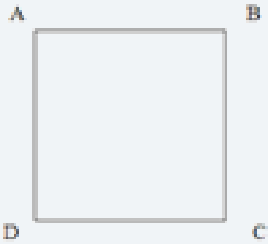
1. Diagonal bisect each other at right angles.

2. All rhombuses are parallelogram but the reverse is not true.

3. A rhombus may or may not be a square but all squares are rhombuses

**(c) Square**

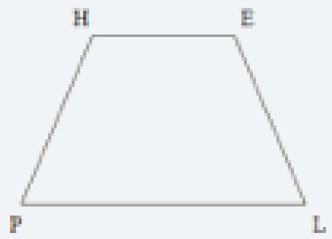
A square is a rectangle with adjacent sides equal or a rhombus with each angle 90.




Area = base× height
Area $= 1/2\ (\text{diagonal})^2$
Perimeter = 4a
Diagonal $= a\sqrt{2}$
Inradius = a/2

**(d) Trapezium**

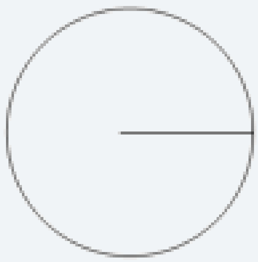
A trapezium is a quadrilateral with only two sides parallel to each other



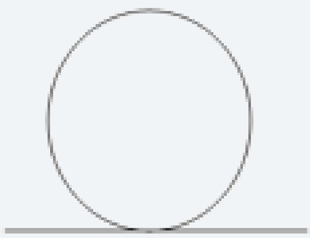

Area =1/2 × sum of parallel sides × height
Median = 1/2 × sum of the parallel sides (median is the line equidistant from the parallel sides).

## CIRCLES

A circle is a curve drawn such that any point on the curve is equidistant from a fixed point.

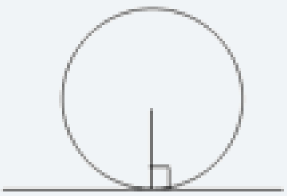


- Diameter is a straight line passing through the centre of the circle and joining two points on the circle .a circle is symmetric about any diameter.
- A chord is a point joining two points on the circumference of a circle.
- Diameter is the largest chord in a circle.
- A secant is a line intersecting a circle in two distinct points and extending outside the circle also.
- A line that touches the circle at only one point is a tangent to the circle

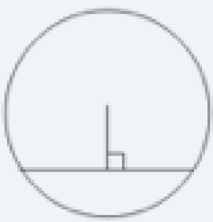


- Two tangents can be drawn to the circle from any point outside the circle and these two tangents are equal in length

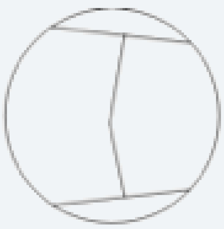
- A tangent is perpendicular to the radius drawn at the point of tangency.



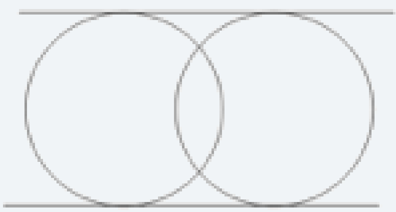
- A perpendicular drawn from the centre of the circle to a chord bisects the chord and conversely, the perpendicular bisector of a chord passes through the centre of the circle.



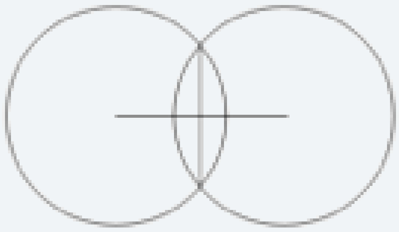
- Two chords that are equal in length will be equidistant from the centre,



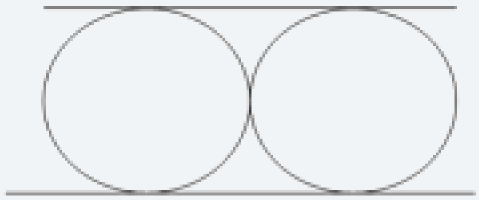
- One and only one circle passes through any three given non–collinear points.



- When there are two intersecting circles, the line joining the centre of s of the two circles will perpendicularly bisect the line joining the points of intersection.



- A tangent drawn common to two circles is called a common tangent.



- Two circles are said to be concentric if they have the same centre. As is therefore here the circle with smaller radius lies completely within the circle with bigger radius.
- Area=$\pi r2$
- Circumference=$2\pi r$
- Area=$1/2 \times$circumference $\times r$

**Arc**

It is a part of the circumference of the circle. The bigger one is called the major arc and the smaller one the minor arc.

Length= $O/360 \times 2\pi r$

- Sector of a circle is a part of the area of a circle between two radii.

  Area of a sector = $O/360 \times 2\pi r$

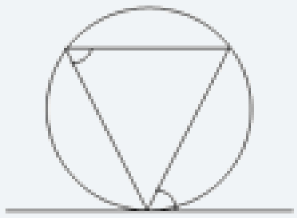
  Segment: a sector minus the triangle formed by the two radii is called the segment of the circle.

  Perimeter of segment = length of the arc + length of the segment

- Congruency: two circles can be congruent if and only if they have equal radii

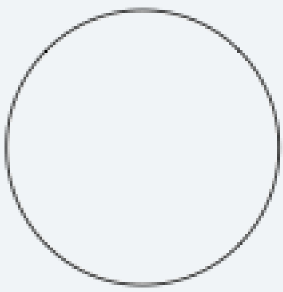
### ALTERNATE SEGMENT THEOREM

The Angle Between a tangent and a cord through the point of contact of the tangent is equal to the angle made by the chord in the alternate segment.



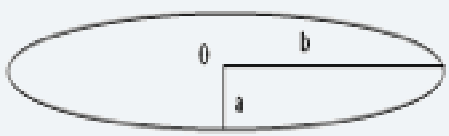
## MENSURATION

### CIRCLES



Area = $\pi r^2$
Circumference = $2\pi r$

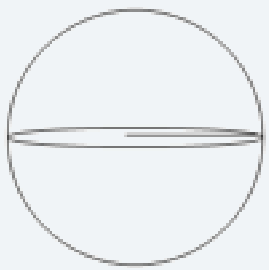
### ELLIPSE




Area = $\pi ab$ where a is semi- major axis and b is semi-minor axis

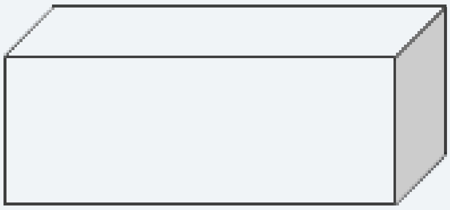
Perimeter = $\pi$ (a+b)

### SPHERE



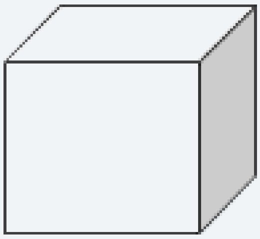
Surface area of a sphere = $4.\pi r^2$
Volume of sphere = $(4/3)\pi r^3$
The curved surface area of a hemisphere is equal to half the surface area of a sphere. i.e. $2.r^2$

## CUBOID



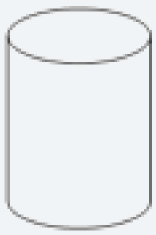
Volume = lbh
Curved surface area = 2( lh+ bh)
Total surface area = 2(lb + lh + bh)

## CUBE



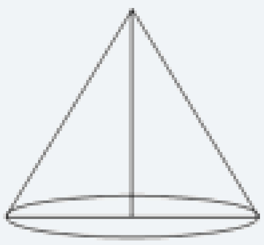
Volume = $a^3$
Curved surface area = $4a^2$
Total surface area = $6a^2$

## CYLINDER



- curved surface of a right cylinder = $2\pi rh$ where r is the radius of the base and h the height.
- whole surface of a right circular cylinder = $2\pi rh + 2\pi r^2$
- volume of a right circular cylinder = $\pi . r^2 . h$

## CONE



- Curved surface of a cone = $\pi rl$ where l is the slant height, r is the radius of the circular base, h is the height.
- Whole surface of a cone=$\pi rl + \pi r^2$
- Volume of a cone=$\pi r^2 h/3$

## CONE FRUSTUM

If a cone is cut into two parts by a horizontal plane, the bottom portion is called the frustum of a cone.

If r is the top radius R the radius of the base: h the height and l the slant height of a frustum of a cone

Lateral surface area of the cone = $\pi l$ (R+r)

Total surface area = $\pi$ ($R^2$+$r^2$ +R.L+r.l)

Volume =1/3 $\pi$ h ($R^2$+ Rr+ $r^2$)

$L^2$=(R-r)2+ $h^2$

If H is the height of the complete cone from which the frustum is cut, then from similar triangles, we can write the following relationship.

r/R = (H-h)/H

## PRISM

A prism is a solid which can have any polygon at both its ends. Its dimension is defined by the dimensions of the polygon at its ends and its height.

- Lateral surface area of a right prism = perimeter of base × height
- Volume of a right prism = area of base × height
- Whole surface of a right prism=lateral surface of the prism + the area of the two plane ends.

## PYRAMID

Pyramid is a solid which can have any polygon at its base and its edges converge to a single apex. Its dimensions are defind by the dimensions of polygon at its base and the length of its lateral edges which leads to the apex.

Slant surface of the pyramid= 1/2 x Perimeter of the base x Slant height

Whole surface of the pyramid= Slant surface + Area of the base

Volume of the pyramid= 1/3 x Area of the base x Height

## Permutations and Combinations

Permutation is the arrangement of things whereas combination is selection of things. Again, in Permutation, 'order' becomes important while 'order' has no significance in combination. e.g. Event of hand-shaking in a party – Combination (order is not important).

Event of exchanging cards in a party – Permutation (order is important).

**Definitions:**

1. An arrangement without replacement is called a permutation. Arrangement of cards, number problems without repetitions are examples of permutation.
2. A selection of n objects without replacement is called a combination.

**Fundamental rules:**

1. The Sum Rule: Suppose a work A can be done in a ways and B can be done in b ways and both cannot occur simultaneously. Then A or B (at least one of them) can occur in (a + b) ways. This rule is also applicable for two or more exclusive events.
2. The Product Rule: Suppose there are two works A and B. Let the work A be done in a ways and the work B in b ways. And the ways of happening of A and B are independent to each other. Then, both A and B can be done in a×b ways

   For example, let there be two works A and B which can be done in 4 ways and 3 ways respectively. Then either of the work (A or B) can be done in 4 + 3 = 7 ways and both the works (A and B) can be done in 4 × 3 = 12 ways.

**Important results :**

1. $n! = 1 \cdot 2 \cdot 3 \cdot \ldots n$; $0! = 1$
2. Number of permutation of n distinct things taken r at a time, $0 \le r \le n$

   $= n(n-1)(n-2)\ldots(n-r+1)$.

   $= n!/(n-r)! = {}^nP_r$
3. The number of permutations of n distinct objects taken all at a time = n!.
4. The numbers of combinations of n objects taken r at a time, $0 \le r \le n$.

   $$\frac{n(n-1)\ldots(n-r+1)}{1.2.3\ldots r} = \frac{n!}{(n-r)!r!} = {}^nC_r$$
5. Number of permutations of n things, out of which p are alike and are of one type, q are alike and are of second type, r are alike and are of third type and rest are all different = $n!/p!q!r!$

6. Number of selections of r things ($r \le n$) out of n identical things is 1.

7. Number of permutations (arrangements) of different things taking r at a time when things can be repeated any number of times = $n \times n \times \ldots r$ times = $n^r$

8. Total number of selections of zero or more things from p identical things = p + 1.

9. Total number of selections of zero or more things from n different things

   $= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$.

10. Number of ways of distributing n identical things among r persons when each person may get any number of things = ${}^{n+r-1}C_{r-1}$

11. **Distribution into Groups:**

a. The number of was in which n distinct objects can be split into three groups containing respectively r, s and t objects, r, s and t are distinct and r + s + t = n, is given by

$${}^nC_r \cdot {}^{n-r}C_s \cdot {}^{n-r-s}C_t = \frac{n!}{r!s!t!}$$

b. If 3n things are to be divided equally between 3 persons (i.e. division of 3n things into 3 equal groups with permutation of groups) then the number of ways

$$\frac{(3n)!}{(n!)^3}$$

c. If 3n things are divided into three equal groups, then the number of ways =

$$\frac{(3n)!}{n!n!n!3!} = \frac{(3n)!}{3!(n!)^3}$$

Since for any one way the three groups can be placed in 3! ways without obtaining the new division. So it is divided by 3!.

12. **The greatest value of ${}^nC_r$**

a. When n is even, ${}^nC_r$ is greatest when r = n/2.

b. When n is odd, ${}^nC_r$ is greatest when r = (n + 1) /2 or (n – 1)/2.

13. If ${}^nC_x = {}^nC_y$, then either x = y or x + y = n

14. ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

## 15. Restricted Permutations and Combinations:

a. Number of combinations of n distinct things taken r at a time when p-particular things always occur = ${}^{(n-p)}C_{(r-p)}$

The reason is that since p particular things are to be taken in each selection, so to make the selection of r things, we are to select only (r – p) things from the remaining ( n – p) things which can be done in ${}^{(n-p)}C_{(r-p)}$ ways.

b. The number of permutations of n distinct things taken r at a time when p particulars things always occur = ${}^{(n-p)}C_{(r-p)} \times R!$

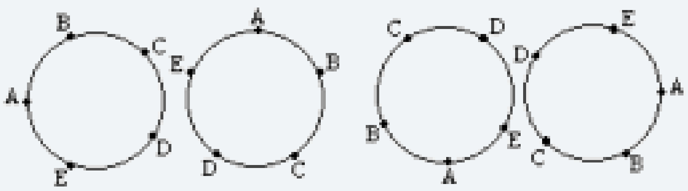
Since number of combinations = ${}^{n-p}C_{r-p}$ and each combination consists of r objects which can be permuted in r! ways. The result follows by the product rule.

c. Number of combinations of n distinct things taken r at a time when p particular things never occur = ${}^{(n-p)}C_r$

d. Number of permutations (arrangements) of n distinct things taken r at a time when p particular things never occur = ${}^{(n-p)}C_r \times R!$

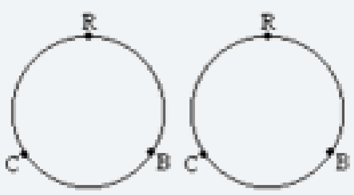
## 16. Circular Permutations:

a. Arrangements round a circular table: A circular table has no fixed starting point or ending point and arrangements like those given below in the figure are considered identical.




If n persons are to be arranged in a straight line, there are n! different ways in which this can be done. When n persons are to sit round a circular table, each arrangement will be repeated n times, so there are (n – 1)! different arrangements.

b. Arrangement of beads (all different) around a circular wire: A circular wire differs from a circular table because when we turn it over we see that the other side presents an arrangement of different beads different from that on the first side.



c. If the wire on the left is turned over we obtain the arrangements on the right. We can see that the only two different arrangements of three coloured beads on a circular wire can be found on opposite sides of same wire, i.e. there is actually one arrangement. More generally, n beads on a circular wire can be arranged in 1/2 (n-1)! different ways.

## Probability

1. Probability is defined as the chance that an event would occur. Hence, the corner stone of probability is the definition of the event.

2. Events are defined with the help of ANDs and ORs and the key to solving probability is the conversion of the Question statement into the Answer statement. The following example would illustrate what we mean by this best:

   There are two boxes, one with 3 red, 4 blue and 5 white balls the other with 4 red, 5 blue and 6 white balls. A ball is drawn at random from one of the two boxes. Find the probability that the ball drawn is white.

   The description of the event above is what we can call as the "Question Statement." The "Answer statement or the Event Definition" of the above question can be defined as below:

   Choose the First Box AND Choose a white ball

   OR Choose the Second Box AND Choose a white ball

   Hence, the required Probability would be:

   P (First Box)× P( White Ball) + P(Second Box) × P (White Ball) = (½)× (5/12) + ½ × (6/15) = 49/120

3. **The Non Event and Its Use in Probability:**

   For every event, we also can define the Non-event which is exactly the opposite of the event. The Non Event is also denoted by E'.

   Thus, n(E) + n(E') = Total Sample Space

   i.e. the sum of the number of occurrences of the Event + the number of occurrences of the Non-Event = The total number of possible occurrences.

   Also, P(E) + P(E') = 1

   Some examples of events and their non events:

   i) In a throw of a die, the chances of getting a number greater than 4. Non-Event: A number Less than or equal to 4.

   ii) In a throw of 2 dice, the chances of getting a number less than 10. Non-Event: A number Equal to or greater than 10. In other words 10 OR 11 OR 12.

   iii) The probability that Rahul passes at least 1 out of 4 exams. Non-Event: He fails all.

   iv) When a coin is tossed 8 times, the probability that heads turns up at least 2 times. Non-Event: Heads turns up once OR Heads does not turn up at all.

4. **Key Definitions of Probability:**

   a. **Random Experiment:**

      An experiment whose outcome has to be among a set of events that are completely known but whose exact outcome is unknown is a random experiment (e.g. Throwing of a dice, tossing of a coin).

   b. **Sample Space:**

      This is defined in the context of a random experiment and denotes the set representing all the possible outcomes of the random experiment.[e.g. Sample space when a coin is tossed is(Head, Tail).

   c. **Impossible Event:**

      An event that can never occur is an impossible event. The probability of an impossible event is 0. e.g.(Probability of the occurrence of 7 when a dice with 6 faces numbered 1-6 is thrown).

   d. **Mutually Exclusive Events:**

      Two or more events are mutually exclusive if they cannot occur together. (E.g. If an even number appears on a die, an odd number would not appear.)

   e. **Equally Likely Events:**

      If two events have the same probability or chance of occurrence they are called equally likely events. (in a throw of a dice, the chance of 1 showing on the dice is equal to 2 is equal to 3 is equal to 4 is equal to 5 is equal to 6 appearing on the dice.)

   f. Exhaustive Set of Events A set of events that includes all the possibilities of the sample space is said to be an exhaustive set of events. (e.g. In a coin toss, Head or tail is an exhaustive list of possibilities.)

   g. Independent Events An event is described as such if the occurrence of an event has no effect on the probability of the occurrence of another event. (If the first 100 coin tosses are heads, there is no change to the fact that the probability of a heads in the 101st throw remains 0.5)

   h. Conditional Probability It is the probability of the occurrence of an A given that the event B has already occurred. This is denoted by P(A/B). (the probability that in two throws of a die we get a total of 7 or more, given that in the first throw of the die the number 5 had occurred)

5. **The concept of Odds For and Odds Against**

   Sometimes, probability is also viewed in terms of odds for and odds against an event.

   Odds in favour of an event E is defined as: P(E)/P(E)'

   Odds against an event is defined as: P(E)'/P(E)

# Set Theory

Set theory is an important concept of mathematics which is open asked in aptitude exams. There are two types of questions in this chapter:

a). Numerical questions on set theory based on venn diagrams

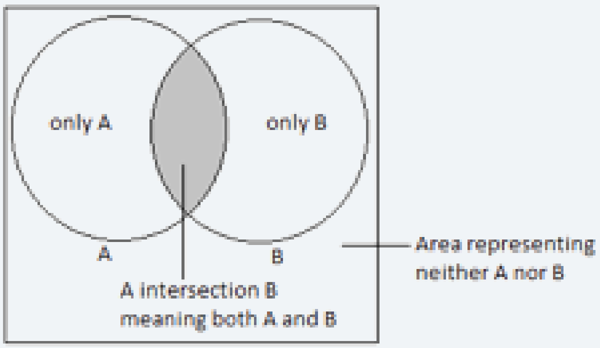
b). Logical questions based on set theory

**Let us first take a look at some standard theoretical inputs related to set theory.**

Set Theory

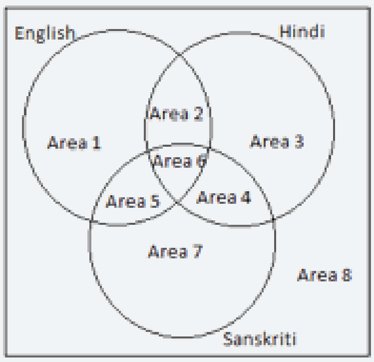
Look at the following diagrams:

**Figure 1:** Refers to the situation where there are two attributes A and B. (Let's say A refers to people who passed in Hindi and B refers to people who passed in English.) Then the shaded area shows the people who passed both in Hindi and English.




In mathematical terms, the situation is represented as: Total number of people who passed at least 1 subject = A + B – A∩B

**Figure 2:** Refers to situation where are three attributes being measured. In the figure below, we are talking about people who passes English, Hindi and/or Sanskriti.



**In the above figure, the following explain the respective areas:**

**Area 1:** People who passed in English only.

**Area 2:** People who passed in English and Hindi only. (in other words-People who passes English and Hindi but not Sanskriti)

**Area 3:** People who passed Hindi only

**Area 4:** People who passed in Hindi and Sanskriti only. (also, can be describe as people who passes Hindi and Sanskriti but not English)

**Area 5:** People who passed in Sanskriti and English only. (also, can be describe as people who passes Sanskriti and English but not Hindi)

**Area 6:** People who passed English, Hindi and Sanskriti

**Area 7:** People who passed Sanskriti

**Area 8:** People who passed in no subjects
Also take note of the following language which there is normally confusion about:
People passing English and Hindi – Represented by the sum of area 2 and 6
People passing English and Sanskriti – Represented by the sum area 5 and 6
People passing Hindi and Sanskriti – Represented by the sum area 4 and 6
People passing English – Represented by the sum of the area 1, 2, 5 and 6

**In Mathematical terms, this means:**
Total number of people who passed at least 1 subject =

$P + C + M - P \cap C - P \cap M - C \cap M + P \cap C \cap M$

## Squares & Roots

| Number | Square | Square root |
|---|---|---|
| 1 | 1 | 1.000 |
| 2 | 4 | 1.414 |
| 3 | 9 | 1.732 |
| 4 | 16 | 2.000 |
| 5 | 25 | 2.236 |
| 6 | 36 | 2.449 |
| 7 | 49 | 2.646 |
| 8 | 64 | 2.828 |
| 9 | 81 | 3.000 |

| Number | Square | Square root |
|---|---|---|
| 10 | 100 | 3.162 |
| 11 | 121 | 3.317 |
| 12 | 144 | 3.464 |
| 13 | 169 | 3.606 |
| 14 | 196 | 3.742 |
| 15 | 225 | 3.873 |
| 16 | 256 | 4.000 |
| 17 | 289 | 4.123 |
| 18 | 324 | 4.243 |
| 19 | 361 | 4.359 |
| 20 | 400 | 4.472 |
| 21 | 441 | 4.583 |
| 22 | 484 | 4.690 |
| 23 | 529 | 4.796 |
| 24 | 576 | 4.899 |
| 25 | 625 | 5.000 |
| 26 | 676 | 5.099 |
| 27 | 729 | 5.196 |
| 28 | 784 | 5.292 |
| 29 | 841 | 5.385 |
| 30 | 900 | 5.477 |
| 31 | 961 | 5.568 |
| 32 | 1,024 | 5.657 |
| 33 | 1,089 | 5.745 |
| 34 | 1,156 | 5.831 |
| 35 | 1,225 | 5.916 |
| 36 | 1,296 | 6.000 |
| 37 | 1,369 | 6.083 |
| 38 | 1,444 | 6.164 |
| 39 | 1,521 | 6.245 |
| 40 | 1,600 | 6.325 |

| Number | Square | Square root |
|---|---|---|
| 41 | 1,681 | 6.403 |
| 42 | 1,764 | 6.481 |
| 43 | 1,849 | 6.557 |
| 44 | 1,936 | 6.633 |
| 45 | 2,025 | 6.708 |
| 46 | 2,116 | 6.782 |
| 47 | 2,209 | 6.856 |
| 48 | 2,304 | 6.928 |
| 49 | 2,401 | 7.000 |
| 50 | 2,500 | 7.071 |
| 51 | 2,601 | 7.141 |
| 52 | 2,704 | 7.211 |
| 53 | 2,809 | 7.280 |
| 54 | 2,916 | 7.348 |
| 55 | 3,025 | 7.416 |
| 56 | 3,136 | 7.483 |
| 57 | 3,249 | 7.550 |
| 58 | 3,364 | 7.616 |
| 59 | 3,481 | 7.681 |
| 60 | 3,600 | 7.746 |
| 61 | 3,721 | 7.810 |
| 62 | 3,844 | 7.874 |
| 63 | 3,969 | 7.937 |
| 64 | 4,096 | 8.000 |
| 65 | 4,225 | 8.062 |
| 66 | 4,356 | 8.124 |
| 67 | 4,489 | 8.185 |
| 68 | 4,624 | 8.246 |
| 69 | 4,761 | 8.307 |
| 70 | 4,900 | 8.367 |
| 71 | 5,041 | 8.426 |

| Number | Square | Square root |
|---|---|---|
| 72 | 5,184 | 8.485 |
| 73 | 5,329 | 8.544 |
| 74 | 5,476 | 8.602 |
| 75 | 5,625 | 8.660 |
| 76 | 5,776 | 8.718 |
| 77 | 5,929 | 8.775 |
| 78 | 6,084 | 8.832 |
| 79 | 6,241 | 8.888 |
| 80 | 6,400 | 8.944 |
| 81 | 6,561 | 9.000 |
| 82 | 6,724 | 9.055 |
| 83 | 6,889 | 9.110 |
| 84 | 7,056 | 9.165 |
| 85 | 7,225 | 9.220 |
| 86 | 7,396 | 9.274 |
| 87 | 7,569 | 9.327 |
| 88 | 7,744 | 9.381 |
| 89 | 7,921 | 9.434 |
| 90 | 8,100 | 9.487 |
| 91 | 8,281 | 9.539 |
| 92 | 8,464 | 9.592 |
| 93 | 8,649 | 9.644 |
| 94 | 8,836 | 9.695 |
| 95 | 9,025 | 9.747 |
| 96 | 9,216 | 9.798 |
| 97 | 9,409 | 9.849 |
| 98 | 9,604 | 9.899 |
| 99 | 9,801 | 9.950 |
| 100 | 10,000 | 10.000 |